VISUAL CALCULUS OR PERCEPTUAL FRIBBLE? WORLD MAPS WITH CONSTANT SCALE EDGES: A NOVEL PROJECTION METHOD, WELL-SUITED TO OUR ERA

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ABSTRACT: I introduce world maps with constant scale edges [cse], and show how cse encompasses both conventional projections and a new class of maps—world maps with constant scale natural boundaries [csnb]. Given context this, apparently, strains mathematics, rejoining a "battle" twixt numbers and geometry thought by some, perhaps, long-settled in favor of algebra. Here geometry betters its cousin, as when numbers, shy the idea of an irrational number, once lost the number-line's challenge.

The idea of "natural boundary" mapping is simple, and in principle offers substantial advantage for studying natural, globular objects—dynamic and spherical planets like Earth and Venus, and odder things like Eros and Ios. But, such maps are difficult to make, and heretofore suffered the usual drawbacks of maps in general, e.g. distorted periphery and unintelligible edge. Computers would end the tedium of plotting points; and constant scale joined to natural boundary re-imagines maps.

I paint a methodology of projection-geometry turned inside out. It works from edge to middle rather than (Mercator, 1569) the other way 'round. This begs no new from software arts. Peripheral distortion yields to peripheral "hinging"; field distortion, unlike customary rubber-sheetings, expresses as shrinkage--balanced along proscribed centrelines.

I trace to Albrecht Durer, and James Clerk Maxwell; I thread through Robert {Lee} Frost, and {Harold Calvin)} Marston Morse. I tell how csnb may, in some situations, be approximated by equal azimuth projections.

I describe csnb's novel properties in general and in plate tectonics by example.

1. OVERVIEW

1.1. Hallmarks of our Era

All scientists are familiar with the fundamental problem of representing curved or complex volumes in two dimensions, as on a map. investigations use stereographic views, which are unexcelled for small regions, but not for proportionate assessment of global events, issues or

ideas--of which these days there's a lot, and a lot known about them. The problem is making sense of lots of data.

1.2. The News:

csnb projections offer a new perspective on, and new tool to probe--processes and structures of irregular solid surfaces.

1.3. Motivation

The rub being that digitizing csnb is a large bump in cartography's road—a big effort of software design, at odds with existing systems, must be done.

2. HISTORY

2.1. some Quotations

"...instead of representing the 'Archimedean' solids in perspective or stereographic images, Durer invented the apparently original and, if one may say so, prototopological method of developing them on the plane surface in such a way that the facets form a coherent 'NET' (Fig. 1.q) which, when cut out of paper and properly folded where two or more facets adjoin, will form an actual, three-dimensional model of the solid short to s.-like continuations of t. high ground of efficiency from geometry. in question." (Panofsky, 1943)

"...452 years ago, at a time when new discoveries had outmoded medieval and ancient maps, and explorations had widened horizons and created a need for better maps...-.Gerard Mercator employed the new principle of perspective to draw up a map which broke with the principles of Ptolemy and which, with later improvement, has been basic to cartography ever since." (Boyer, 1968)

"Hence the whole earth may be naturally divided into...Dales, and also, by an independent division, into Hills, each point of the surface belonging to a certain dale and also to a certain hill." (Maxwell, 1870)

"...in mathematics there are those who would like to keep algebra and geometry apart, or would like to subordinate one to the other. The battle became acute when the discovery of analytic geometry by Descartes made it possible to represent all geometry by algebra. The battle between geometry and algebra has been waged from antiquity to the present." (Morse, 1950)

2.2. Discussion

If that's history, that's also a cryptic leapfrog; but space is dear.

2.2.1. Durer: If any art historian could say *prototopological* about Durer, it'd be Princeton's Panofsky. Marston Morse, who wrote the book on topology--A

Calculus of Variation in the Large, a k a Morse Theory--was Panofsky's colleague at the nearby Institute for Advanced Study.

2.2.2. Mercator: Bover describes the birth of modern cartography. Mercator's invention was soon re-imagined as formula (Wright, 1615), numbers seizing

Figure 1: s. folds up to t. A q.-like r. may be fashioned. but will fall

> **2.2.3. Maxwell:** Maxwell's genius for descriptive language sets the stage for Morse. His "independent" dales and hills are delineated by lines of watershed and watercourse.

hills

These lines are overlapping systems, and form a grid—a net of strings. This is the vocabulary of cse/csnb.

Investigators vary their images by stepping from one grid to the other at points (passes and bars to Maxwell, just passes to Morse) where the grids intersect.

These lines may be any wiggly set one wishes, regardless of altitude: tectonic activity, the planet's largest lines, for example, or meteorology, our most fluid lines.

Maxwell's ups and downs have since become fundamental to physics—the "rolling ball" of gravity and electricity. This expands cse/csnb's scope.

A Molecular Mapping Group (Autschbach, 2000) could make quick use of csnb. Carrying, without regard for a more factual way, the familiar illusion of Mercator into electron-density, merely because csnb is late to the gate, is correctable.

2.2.3.1. Examples: The borders of Figs. 5, 13 & 14 stay not only in Maxwell's topography, but also on watersheds. Fig. 4's upper images use watersheds and watercourses (current divides) as map edge, stepping from one net to the other where land meets ocean.

Trenches, in the status of "centreline," make for Fig. 7.

2.2.4. Morse: Morse was not one to give the battle up.

Raoul Bott, his apprentice and later his biographer, said, "[Morse] was in a sense a solitary figure, battling the algebraic topology, into which his beloved Analysis Situs had grown."

2.3. World Maps with Natural **Boundaries**

The idea could be old-- da Vinci made a cloverleaf-shaped map that pits, peaks {should} fold up, and the subject was and passes

explored mid-20th century (Spilhaus and Snyder, 1993; Mulcahy, 1999). And, ancient, irregular map "scraps' appear in the light of csnb curious too, as the idea is easy to stumble upon.

2.4. World Maps with Constant Scale Natural Boundaries

A special case of world maps with constant scale edges.

Durer's drawings express planar facets. Durer didn't lay out the unfolded version of Fig.1.r. but, given q., that may be imagined; compare it with s., the same form--a human head.

Durer's cue is an exhaustion of planes, csnb's from Maxwell's strings. Both methods approach the ideal, but csnb gets there faster, so to speak.

3. CONTEXT

Figure 3: rubber sheets versus Hinging.

3.1. Reactions to cse/csnb

"Continental divides for a map's boundary, you did it? I thought of that...couldn't figure out how, though....Centrelines, huh? I don't know what you mean. Ya got a formula?...No, too bad, they won't listen to you," said Athel Spilhaus, along with "I'm eighty-six today, ya know..." when I called to discuss a map I'd sent him.

"Oh, these are just D'Arcy Thompsonisms..." said cartographer Keith Clarke, in response to similar maps (Thompson, 1916).

"There is at present no software of any kind that supports any sort of maps with

natural boundary, especially [csnb]. The idea is so odd, there's no way to compare [it to conventional projections]....Sure, in principle natural boundary maps have intriguing advantages, but they suffer conventional So, with new, exciting flaws. **b.** stereography, why bother [with strict watercourses cartographic product]? ... And, no one's asking for it. If we had re-

and passes quests for natural boundary maps, we'd start making them....On the other hand, that's a difficult thing to do. Scissoring conventional projections, like Spilhaus, is extremely complicated. [csnb] is so different, there's telling..." (Mulcahy, 1999)

Figure 2: chart of an inland basin,

rendered.

composite of b+

dales

Watersheds,

and pits

Maxwell's as published, In 2000, the ACSM Map 1870 Competition 2000 Design declined to premiate Figs. 13 and 14, entered in the category *other*, as "projections." The jury declined to open

the discretionary *other* category.

Bott, on reviewing csnb informally, says he sees nothing in cse/csnb of Morse Theory.

Christensen, who defines himself as an analytic cartographer, says about csnb I see no numbers there.

3.2. Comment

Apparently, cse/csnb exploits something—a mutual normality betwixt independent divisions, perhaps--in Maxwell's work untouched by Morse.

4. RESPONSE

Apprehension has always been the lure, the charm of maps. It's remarkable that a demonstrably appropriate cartographic system --indeed any cartographic projection system, much less one Mercator could understand--is not digitized; remarkable save that maps are hobbled by centuries-old flaws, marginalized by digital stereography, and outpaced by analytic discoveries (topology). That strict cartography--a "mature" discipline--has the fitness for a great leap surprises.

> That cse/csnb is not a D'Arcy Thompson-ism (Fig 3-top) is easy to see: his sheet's edge, which varies in length, is nonexistent in csnb. There is, of course, in csnb a reduction of area, 3-D to 2-D, but the anology is to the repair with a heat gun of a bulge in a mylar mirror, rather than the stretch of a rubber sheet.

The process is anamorphic sculpture rather than anamorphic drawing (Clark 1980). See Fig. 17.

Does loose cartography play to the eye's strength? Yes, but only of the extent the eye That's a limited arena, when global



components play. Stereography never shows in one view all the surface of an essentially globular object.

Krantz puts well the present case for maps in a hot-image world (Krantz 1999). In addition, csnb permits regions defined by Maxwell's terms—worthwhile distinctions in principleto be seen in entirety: Fig. 6 is the basin of Africa and Asia; Fig. 6 is the Antarctic basin.

A common map use is as an adjunct to reasoning. Once a pattern is recognized, the map is needed not, though no less enjoyed. Figs. 5 & 6, like an on-axis view of a motor, show how earth's spin drives ocean, currents. Yet Fig. 6 displays a subtlety obscure in Fig. 5. The currents clearly have two "ears"—Antarctica's great gulfs--each associated with a strong, northward, basement current.

Is analysis flawed? No, just incomplete. those Poincaré and Morse, most responsible--Morse might say "at fault"-- m. for analysis' advanced state, went to lengths to show that it is but one side of thinking's coin.

Figure 4: Zipping, follow the arrows and watch the continents coalesce

Or, hinge-arcs may be set so the enclosed area is at a maximum, which maximizes conformality and produces a folded model fitting an egg.

Hinging is shown in Fig. 3.

6. PROPERTIES

6.1. Maps can be folded up to 3-D replicas

Antipodal geometry is preserved; maps' antipodes are conformal, unlike usual maps. And, unlike geodesic maps (which also fold), csnb's antipodes are naturally bounded, which begets geometry germane to inquiry. (Spilhaus, 1987)

For spheres, globes are the only true representation of the body, but for extremely irregular solids like Eros, or extremely irregular processes like tectonics, csnb maps can in principle be transformed to solids.

And, sheets could be economically printed with photos, then folded into photo-real models.

6.1. leads to 6.2.

5. METHODS

Boundaries never change in length, only in azimuth; boundaries define regions, and hence centrelines of regions. Centrelines change only in length, never in azimuth. If these restrictions were relaxed, and the edge could be any imaginary line (such a line of longitude) and could stretch, then conventional, formula-based projections—all of them--would result, as a little thought will show.

Fig. 2 illustrates the sophistication of Maxwell's independent divisions. Compare the many patterns.

Figure 5: a world map bounded by the South Pole

5.1. Zipping

Fig. 4's progression could continue in the zero direction, i.e. beyond **m.** & **n.**, to the limit of a single point, a mountaintop in Malaysia, Fig. 15, for example, where one crosses to the world of conventional projections, in the style of an equal azimuth (Unknown, c2000BC) which breaks the sphere at a single point. This suggests something—a cross-over spot between csnb and convention projections. See section 7.

Conversely, the progression may reverse towards another crossover point, conventional orthography

Figs.4.m. & n. enlarge as Figs.13.& 14.

5.2. Hinging

Changes in azimuth occur at hinges. It is not immediately clear that overlaps may always be eliminated, but adding hinge-points and altering the hinge-arcs has always worked.

Hinge-arcs may be set so they are as equal as possible, that minimizes enclosed area, and gives a folded bounded by valleys of Antarctica

6.2. Maps can be "zipped up," and still folded

advantage--large-region unification-follows from **6.1.** It speaks to csnb's quiddity: these maps may be zipped completely, or completely unzipped--zero and infinity, the basics-without loosing their "prototopological" nature.

6.3. Maps can represent solids with holes or deep fissures

It is difficult to construct a global weather map because of upand down-welling between stratified altitudes. csnb can handle such problems. Another application, in physical chemistry, might be conformal representation of electron-density. And, in forensic anthropology, csnb might be helpful in identifying tribal affinities for museum-remains repatriation. (Clark, 2002)

6.4. Maps can compare multiple global parameters

In geophysics, csnb might be useful for showing seismic tomography; in biology, for tracking nuances in cell motion, or depicting with precision cell division.

In Fig. 6, proportionate regional differences—laps and gaps--between glacial watersheds and underlying rock watersheds could be clearly compared, without loosing focus on the rest of Earth's surface.

In meteorology, stalled hurricanes could be as clear, leading to better weather predictions.

6.5. Maps permit geometric harmonic analysis

This is useful for exploring natural boundaries of force or motion. A Medial Axis, described by waterlining, locates centroids, directs vectors, and assists in measure of distance (Christensen, 1999). See Section 8.

6.6. Highly zipped maps have curious characteristics

When csnb maps are folded up they have weak and strong axes, which may mimic an object's actual conditions, such as at Earth's trenches and spreading ridges.

At molecular orders of volume, this feature promises intriguing models. It implies that, in some sense, a molecule of, a case, hydrogen chloride, modelled in csnb, *inhabits* the solid modelled by the folded map.

csnb works well in pairs—maps with independent, complementary viewpoints. See Figs. 13 & 14.

Figure 7: lithospheric stress patterns

7. SUGGESTION

csnb's effectiveness may be tested at Great Points--spots with global consequences or causes, such as big impacts, eruptions, storms, and tidal anomalies—because point-event csnb is logically synonymous with *equal azimuth*, which breaks the sphere at a single point. At relatively little effort, csnb may be tried by investigators.

Contemplate the currents: Fig. 6. has 5.'s data and a readable edge---Antarctica's valleys.

Why it's a good idea not to put map-midst a Great Point is counter-intuitive. Received wisdom says "centre," but this puts an interesting part at the usually obscure periphery. Yet in csnb, this displays as a single, potentially pattern-revealing, shape.

Compare Antarctica's two bays, obscure in Fig. 6., but proportionate in Fig. 5. See Fig. 16 for Key.

Maps of Chicxulub and Pinatubo, two famous Great Points, may reveal little, because where aerosols enter stratosphere has little effect on results—it's the insignificance of a gas nozzle in a chamber—it all blends around the equator. However, such effects may be noticeable on bodies without atmosphere.

And, if Tarawa-edged maps make a pattern of tidal nodes, this may help with other, long-rumored anomalies—*great waves* off South Africa, and *maelstroms* off Norway—that might have the same interplanetary cause.

In meteorology, a statistical tie of Malaysian atmospherics and Atlantic hurricanes is also, a pictorial possibility. See Figs. 14 & 15.

8. EXAMPLE, PLATE TECTONICS

8.1. Summary

If the "orientation of the intraplate stress

field is largely controlled by the geometry of the plate boundaries," and by "compressional forces," then these stresses may be depicted, animated, and refined by a *Wave Test* on a csnb map. It acts as a sort of graph paper, similar to a 19th century-science wave pool.

The map's boundary generates an initiating wave. Interactive animation—interference resulting from a Medial Axis--permits visual appraisal of tectonic stress patterns. (Zoback, 1992, Christensen, 1999)

And, insight may be gained haptically, by toying with *Mechanical Globes* constructed from csnb.

8.2. the wave test, beginning

An appropriate map configuration is reached by zipping and hinging until the resulting "first-order, mid-plate, uni-axial stress vectors" best fit the data. To some extent, especially in a new science like tectonics, this is a subjective choice. The shape is of contemplative benefit in its own right (Lowman, 1999).

For example, Fig. 7 shows at the map's boundary a subset of spreading ridge. Moments may be summed about a point; arrows may collect in a minimum circle; or, something analogous to a Maxwell String Diagram could be examined for equilibrium--tweaks---in the edge.

First-order stresses express as wave fronts (waterlines) inwards from map edge, and second-order stresses as functions carried on the first-order waves.

A map-user, finer tuning the edge to best fit image with experiment, so balances the "ratio of first- and second-order magnitudes," and finds that a "careful evaluation of observed local rotations" has been done.

On one figure, this is very "careful evaluation."

8.3. the wave test, continuing

Waterlining establishes the map's Medial Axis—a 2-D polyvector—that may then be merged with fractures, faults and trenches to make a new Medial axis, slightly out of harmony with the map's edge.

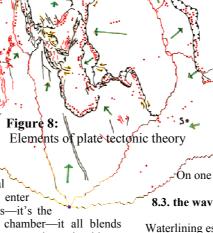
The process reverses. A new wave front projects outward from the medial mean, retuning the edge as investigators deem, or

experiment dictates. Coulomb stresses can be incorporated, Christensen demonstrates, by varying the x y ratios of the wave fronts.

Because of its edge, Fig. 8 (a simplified Fig 7) remains readable without coast- or sialic-lines.

An opposite map from Fig. 7 would put tectonic's least certain portion (continental fracture piles) at csnb's most accurate area—the edge. A *wave tets* on

this might give a novel, and careful, evaluation of these fuzzy zones. Stress shadows might be clear.



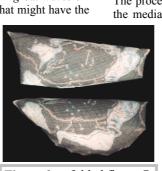


Figure 9: a folded figure 7.

Where the test fails to follow facts, or one seeks prediction, various scenarios of back- and forecasting could adjust the image; or would, save that csnb does not, in the world of conventional projections, exist.

8.4. Mechanical Globes

Ridges are interlaced, whence manipulable models—pull on the Indian Ocean triple junction and trenches in Indonesia and India subduct.

The meaning, if any, of Fig. 9's condensed Figure 10: a map zipped form is unclear; but (if mathematics relates to nature) the "prototopological" quality of csnb midway Figs. 7 & 11 must mean something; a puzzle, tantalizing to those who--as Morse put it in a birthday greeting to Robert Frost--enjoy, and endure the joys, of entertaining the unknown. (Morse, 1959)

At an intermediate level of zipping (imagine a coarsely peeled apple) spherical divergence gives folded model strong and weak zones, areas which correspond (perhaps only loosely correspond, but...) to each plate's uni-axial influence, the first-order intraplate stress field. See Fig. 10.

At minimal levels of zipping, maps and models could be handy explaining basic concepts See Figs. 11 & 12.

Such models may be helpful for understanding "uncertainties such as hydrostatic pore pressure" and "relative magnitudes of intermediate principal stresses." Perhaps only glimpses, but such glimpses could be plugged back into further wave tests...

8.5. Example enlarged

This same sort of cartographic depiction process, applied here to very slow dynamics—tectonics—could be done in much faster meteorological dynamics; or much more complexity as swarms of asteroids, or molecular chemistry.

9. COMPENDIUM

An infrequent use of maps is as an actual reasoning method—a tool, like in orienteering, and Mercator's map, showing sailors' rhumb lines as straight. Others have explored conventional world maps with natural boundaries (Spilhaus 1991); csnb has an additional quality of an accurate edge, which admits of novel actual uses: a "global orthography" of additional study tactics.

csnb's proportional fidelity and intelligible perimeter--an edge

with exact content--also plays to the eye's strength and, up a level, to the mind's strength. We are just not used to thinking of things in a globally proportionate way. Wild-shaped image? Deal with it. What is, rational string (presumably, or...change the boundary!), a calculus of locales.

Tectonicists, discussing regions, don't say

such and plate-shape such

anymore than astronomers would say some galactic object is too bizarre to contemplate. csnb merely collects that chaos, that reality, within a border more definite than other systems--however extravagant the result.

strict cartographic product, csnb admits of precision both the equal of convention—the middle is just fine—and the better, because csnb's periphery is accurate and intelligible.

csnb complements--perhaps perfectly-stereography's excellent, partial views.

> csnb reduces from three, or conventional best two, to **one** the map number needed for a full look at something.

csnb brings--within a boundary's context---proportionality to global geometry, without degrading a map's middle regions.

> With csnb, insight improves as patternrecognizing capacity increases; and communications improve as images are limited.

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11. APPENDIX

11.1. Complementary map-pairs

Section 8.3 describes another type of map-pair, tectonic twins.

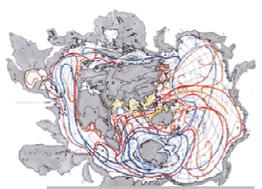


Figure 13: the basin of the Americas.

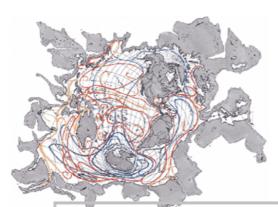
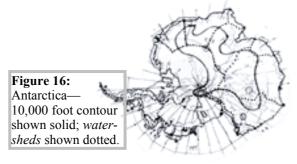


Figure 14: the basin of Africa and Asia

11.2. A Hurricane-chasing map?



11.3. Key for Figs. 5 & 6.



11.4. Anamorphic Sculpture.



Figure 17: Anamorphic sculpture is one dimension more involved than anamorphic drawing.

12. ACKNOWLEDGEMENTS

Poet Robert Lee Frost arranged this toboggan at the top of the hill over 50 years ago; cartographer Ralph Freeman, U.S. Army Corps of Engineers, Jacksonville District, broke it free in 1964; architect (and Houston native) jim hagan gave it an needed nudge in 1990, and continues to contribute. I am grateful for the ride.

FlexMag Industries, a Group Arnold company in Marietta, Ohio, graciously contributed magnetic sheeting, used in hand drafting these maps.

Anonymous reviewer helped with text and tectonic specifics.

Extraterrestrial Mapping Workshopper Randy Kirk encouraged this paper. For the curious (from Facts on File, and other sources), the word "extraterrestrial" was first recorded, and perhaps invented by H.G. Wells, c1900. The first American use "...on the form that intelligent extra-terrestrials might have..." was by L. Sprague de Camp, Design for Life, Astounding Science Fiction, May 1939, Street & Smith Pub., NY.